

## A Study Of The Main Solar Wind-Magnetosphere Energy Input Functions

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**Abstract-** The determination of the energy transferred to the magnetosphere is still a subject for discussion and analysis. Several works tried to obtain a function of the solar wind parameters for approximating as much as possible the energy input to the energy consumed in the inner magnetosphere. After being injected, the energy is accumulated in the magnetotail, where is finally dissipated to different magnetospheric regions. In the present study, the main of these injection functions are bibliographically reviewed. One of the most widely used function is derived, the so-called epsilon parameter of Akasofu. This function is a result of some approximations and empirical values. Our attempt is to reconsider this function with two corrections, both dependent on the solar wind ram pressure: the geometry and the reconnection efficiency.

**Palavras-chave:** solar wind, magnetosphere transfer, energy input functions, magnetic reconnection

**Área do Conhecimento:** Space Geophysics

### Introduction

The solar wind is the main source of energy for the geospace. When the supersonic flow reaches the Earth's environment, embedded in the Interplanetary Magnetic Field (IMF), an interaction between the IMF and the geomagnetic field takes place. The factors controlling this interaction between these fields are not completely understood, but when the IMF z-component is southwardly directed, the annihilation (reconnection) of the geomagnetic field at the dayside magnetopause allows part of the solar wind energy to flow into the cavity (DUNGEY, 1961). Furthermore, momentum and plasma are also transferred to the magnetotail. The efficiency of the three fluxes related to the magnetospheric effects depends on the solar wind parameters, mainly on the Interplanetary Magnetic Field (IMF) direction related to the geomagnetic field.

The solar wind momentum flux, which determines the magnetosphere scale size by the pressure balance, is primarily considered as being  $Nmu^2$ , with  $N$ , the solar wind density,  $m$ , the particles' mass, and  $u$ , the solar wind speed. On the other hand, the magnetic and thermal energies fluxes are given, respectively, by  $uB^2/2\mu_0$ , where  $B$  is the IMF strength, and  $1.5NkTu$ , where  $k$  is the Boltzmann constant, and  $T$  is the solar wind temperature. Based on these three fluxes, the total energy flux that through the magnetospheric frontier might be estimated (KING, 1986). Since the IMF direction and the total energy

and momentum, which blows against the magnetosphere, are known, it is possible to estimate the energy transferred to this cavity formed by the geomagnetic field.

Inside the magnetosphere, the energy is responsible for a variety of processes in different regions. The energy is first stored on the magnetotail and reconverted into primarily thermal mechanical energy in the plasma sheet, auroral particles, ring current and Joule heating of the ionosphere.

An important goal of solar-terrestrial physics is to understand how the rate of solar wind-magnetosphere energy transfer depends upon interplanetary and magnetospheric parameters. The purpose of this paper is to review some of the most important coupling functions used to estimating the energy input.

### Most Common Used Coupling Functions

There are several empirical functions developed for estimating the energy injection into the magnetosphere. Some of them, specially in the beginning of the studies of solar wind-magnetosphere coupling, considered just simple correlations between the main interplanetary and geomagnetic parameters ( $B_z$  component and the magnitude  $B$  of the IMF, the solar wind speed  $u$ , and so on). They tried to establish connections between these parameters and the geomagnetic response. As the studies improved the understanding of some complex phenomena, new formulas started being developed.

Gonzalez et al. (1990) showed that the most widely used coupling functions that correlates well the solar wind and magnetospheric dissipation parameters can be derived as particular cases of general expressions for the momentum and energy transfer at the magnetopause due to large-scale reconnection. In another work, Gonzalez et al. (1994) listed the most commonly used coupling functions to estimate the energy balance into the magnetosphere.

Gonzalez (1973) suggested an expression for the total power input to the magnetosphere due to reconnection process (DUNGEY, 1961) at the dayside magnetosphere as the following expression:

$$\begin{aligned} P_W(S, \theta)(W) &= \frac{8}{\mu_0} u R^2 B_T F(S, \theta) | \vec{B}_G - \vec{B}_M | \\ &= \frac{8}{\mu_0} u R^2 B_T B_M W(S, \theta), \end{aligned} \quad (1)$$

where  $u$  is the solar wind speed;  $R$ , a scale length of the order of the dayside magnetopause radius;  $B_T$ , the transverse (to the Sun-Earth line) component of the IMF, given by  $B_T = (B_y^2 + B_z^2)^{1/2}$  in GSM coordinate system;  $\vec{B}_M$ , the magnetosheath field at the magnetopause;  $\vec{B}_G$ , the geomagnetic field at the magnetopause; and  $F(S, \theta)$ , a function that describes the projection of the magnetosheath electric field to the reconnection line given by Gonzalez and Gonzalez (1981) as  $F(S, \theta) = (1 - S \cos \theta) / (1 + S^2 - 2S \cos \theta)^{1/2}$ ,

with  $\theta$  being the angle between  $\vec{B}_G$  and  $\vec{B}_M$  at the nose of the magnetopause and  $S \equiv |\vec{B}_G - \vec{B}_M| \geq 1$ . From the definition of  $F(S, \theta)$  and Eq. (1),  $W(S, \theta) = (1 - S \cos \theta)$ .

For the limiting case, with  $S = 1$ , Eq. 1 is reduced to:

$$P_W(S, \theta)(W) \cong \frac{2}{\mu_0} u R^2 B^2 \sin^2\left(\frac{\theta}{2}\right), \quad (2)$$

where  $B$  is the IMF magnitude, and  $B^2 \cong (4/\mu_0) B_T B_M$  (GONZALEZ AND GONZALEZ, 1981).

The rate of energy transfer at the frontside magnetopause, due to a large scale reconnection process for an arbitrary interplanetary magnetic field, was considered by Gonzalez and Mozer (1974) to be equal to the Joule heating rate at the reconnection electric field and the magnetopause

current. Since these parameters are represented by parallel vectors, the region involved in reconnection is dissipative.

Perreault and Akasofu (1978) and later on Akasofu (1981) have shown that this power input to the magnetosphere is well described by the coupling parameter  $\varepsilon$ . This parameter was obtained by the reconnection power  $P_K$ , based on the dawn-dusk component of the electric field responsible for the reconnection at the dayside magnetopause. The general expression of the power injected is given by:

$$P_K(S, \theta) = \frac{16\pi}{\mu_0} u R^2 B_T B_M K(S, \theta), \quad (3)$$

with

$$K(S, \theta) = (1 - S \cos \theta)(S - \cos \theta)^2 / (1 + S^2 - 2S \cos \theta)$$

With the same approximation as the one for Eq. (2), Eq (3) is reduced to:

$$\begin{aligned} P_K(1, \theta)(W) &\cong \frac{4\pi}{\mu_0} u R^2 B^2 \sin^4\left(\frac{\theta}{2}\right) \\ &= \varepsilon, \quad \text{with } R \equiv l_0. \end{aligned} \quad (4)$$

The factor  $l_0$  is an empirically determined scale factor with the physical dimension of length,  $l_0 = 7R_E$  (PERREAULT AND AKASOFU, 1978; AKASOFU, 1981), which represents the magnetopause radius.

When  $S = 1$ , the difference between  $K(S, \theta)$  and  $W(S, \theta)$  is maximum. As  $S$  increases, this distinction diminishes, becoming less noticeable until it reaches a limit of large  $S$ ,  $K(S, \theta) \rightarrow (1 - S \cos \theta) = W(S, \theta)$ . Physically, this limit refers to cases when  $B_G$  is sufficiently larger than  $B_M$ , for which the reconnection line does not "tilt" much. Therefore, the dawn-dusk component of the reconnection electric field,  $E_y$ , does not differ much from the total field (GONZALEZ AND GONZALEZ, 1984). Gonzalez and Gonzalez (1981) studied different examples considering different values for  $S$ . And they found that  $P_K(S > 1)$  describes better the power input to the magnetosphere.

Vasyliunas et al. (1982) obtained a more general expression for the magnetospheric dissipated power from the solar wind based on the MHD process:

$$P_W \cong M_E^2 \mu_0^{1/3-\alpha} \rho^{2/3-\alpha} u^{7/3-\alpha} G(\theta), \quad (5)$$

where  $M_E$  is the strength of the Earth's magnetic dipole moment,  $\rho$  is the solar wind density, and  $G(\theta)$  is a dimensionless function dependent on the transverse component of IMF ( $B_T$ ) orientation. If we consider the Chapman-Ferraro scale length  $L_{CF} = (M_E^2 / \mu_0 \rho u^2)^{1/6}$ ,  $M_E^{2/3}$  can be replaced by  $L_{CF}^2$  in Eq (5):

$$P_W \cong \mu_0^{2/3-\alpha} L_{CF}^{2\alpha} B_T^{2\alpha} \rho^{1-\alpha} u^{3-2\alpha} G(\theta),$$

With this assumption, we consider that Eq. (6) is based upon the assumption that the energy transfer rate has a power law dependence on the upstream Alfvénic Mach number,  $M_A^2 = u(4\pi\rho)^{1/2} / B_T$ , with a slope of  $2\alpha$ .

Since  $\alpha$  assumes different values, different expressions are obtained for Eq. (6), and they represent different dependences on solar wind quantities:

$$\begin{aligned} \alpha = 1/2 &\Rightarrow P = \mu_0^{1/6} L_{CF}^2 (\rho u^2)^{1/2} u B_T G(\theta) \\ \alpha = 1 &\Rightarrow P = \mu_0^{-1/3} L_{CF}^2 u B_T^2 G(\theta) \end{aligned} \quad (7)$$

where  $\rho u^2$  is the solar wind ram pressure. If  $L_{CF} = l_0 = \text{cte.}$ , and  $G(\theta) = \sin^4(\theta/2)$ , the first expression of Eq. 7, with  $\alpha = 1/2$ , is equivalent to the widely discussed coupling function  $\epsilon$ . On the other hand, the second expression ( $\alpha = 1$ ) (Eq. 7), with the same expression for  $G(\theta)$  is the coupling function studied by Bargatze et al. (1986) and Gonzalez et al. (1989) in connection with the geomagnetic AL and Dst indices. When  $G(\theta) = 1$  and assuming constant values for  $L_{CF}$  and  $\rho$ , both the expression shown in Eq. 7 are reduced to the respective simple coupling functions  $u B_T^2$  and  $u^2 B_T$ . When  $B_T$  is reduced to  $B_z$ , the simplified versions  $u B_z^2$  and  $u^2 B_z$  are found (BAKER et al., 1981). Later, Monreal-Mcmahon and Gonzalez (1997) took into account the magnetopause position as a function of the solar wind ram pressure by replacing the constant value  $l_0$  by  $L_{CF}$ , so the magnetopause boundary was allowed to change its position following the dynamic pressure variations:

$$\epsilon^* = \left( \frac{L_{CF}}{l_0} \right)^2 \epsilon \quad (8)$$

With this assumption, the effective area was changing its shape, so that the more pressure was put against the magnetosphere, the smaller the effective area would become, because of the compression of the magnetospheric cavity.

As many authors have shown in their works, there are a lot of assumptions to approaching the real energy injected into the magnetosphere and the one afterwards dissipated. There is still a variety of suggestions proposed for improving the epsilon parameter.

## Results

According to the dimensional analysis proposed by Vasyliunas et al. (1982), the most widely used coupling functions  $\nu B_S$  and  $\epsilon$  are related to the power input expression  $P$ , represented by Eq. (6), through the two expressions in Eq. (7). Since the dependence is assumed to be linear, the value  $\alpha = 1/2$  shows a dependence on the solar wind ram pressure as  $(\rho u^2)^{1/6}$ , whilst for  $\alpha = 1$ , there is a dependence on the Chapman-Ferraro scale length. This represents that the dependence related to the dynamical pressure is given as  $(\rho u^2)^{-1/3}$ .

The largest solar wind ram pressure variations are believed to modulate the reconnection process, which seems to be important in the magnetospheric coupling.

Since the following 20 years, the Akasofu's epsilon parameter has shown a very practical way of determining the power input, and it has been widely used on the bibliography. However, there is a variety of doubts related to the physical meaning of this parameter.

De Lucas et al. (2007), following the suggestion of Koskinen and Tanskanen (2002) for a new improvement in the power estimate, and the dimensional analysis of Vasyliunas et al. (1982), considered a new correction on the corrected epsilon parameter by the geometry (Eq. 8), at this time dependent on the dynamic pressure:

$$\epsilon^{**} = \left( \frac{P_{SW}}{P_0} \right)^{1/n} \epsilon^* \quad (9)$$

where  $\overline{P_0}$  is the solar wind ram pressure average taken from a reasonably long time interval. The exponent  $1/n$  (Eq. 9) is considered as  $1/2$  as the increase on the reconnection efficiency is proportional to the  $1/2$  power of the solar wind pressure due to the zero-order balance at the magnetopause (Vasyliunas et al., 1982).

After assuming this, de Lucas et al. (2007) showed that the influence of the ram pressure on

the acceleration of the reconnection process improved the energy estimate.

## Discussion

During the present work it was possible to observe the evolution of the power input estimate since earliest 1970's of the last century. The attempts for obtaining the most approximated values for the energy, dissipated from the solar wind into the magnetosphere, showed that the determination of this power is nontrivial matter. There are several different ways of determining the energization of the magnetosphere. However, most of them do not consider the fundamental parameters with a true physical application. It is also possible that internal magnetospheric parameter need to be incorporated, in order to improve the estimate of the solar wind energy that is transferred to the magnetosphere.

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